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## ALGORISMUS IN GKS 1812 4to

### *Transcription and Translation*

#### Introduction

THE *ALGORISMUS* is an Old Norse prose translation of the *Carmen de Algorismo* of Alexander of Villedieu (c. 1170–c. 1240). The two earliest witnesses to this translation are found in AM 544 4to, a part of Hauksbók, and GKS 1812 4to. These versions are very close to one another and, possibly, to the original, perhaps being only once or twice removed.<sup>1</sup> Notwithstanding the similarities, the orthography and context of the copy in GKS 1812 is notably different from that of the Hauksbók copy. Whereas previous transcriptions of the *Algorismus* have been based on the version in Hauksbók, this paper provides a transcription of the *Algorismus* as it appears in GKS 1812, as well as an English translation.

The *Carmen de Algorismo* is a rendering, in Latin hexameter, of the techniques for working with the Indo-Arabic system of writing numbers.<sup>2</sup> It is itself a descendant of twelfth-century Latin translations of a ninth-century Arabic treatise written by Muḥammed ibn Mūsā al-Khwārizmī (c. 780–c. 850).<sup>3</sup> This work of al-Khwārizmī, known by its Latin title, *De Numero Indorum*, introduced Western Europe to the techniques of arithmetic which came to be referred to as the Indian calculus or *algorism* (derived from the author's toponym). Written in the first half of the thirteenth century, the *Carmen de Algorismo* helped spread the methods of

1 Kristín Bjarnadóttir and Bjarni V. Halldórsson, “The Norse Treatise *Algorismus*,” *Actes du 10ème Colloque maghrébin sur l’histoire des mathématiques arabes* (Tunis: Association Tunisienne des Sciences Mathématiques, 2011), 69, and “*Algorismus*: Hindu-Arabic Arithmetic in GKS 1812 4to,” *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, eds. Gunnar Harðarson with Christian Etheridge, Guðrún Nordal, and Svanhildur Óskarsdóttir (forthcoming), 189.

2 The *Carmen Algorismo* appears in *Rara Mathematica* on pages 73 to 83.

3 For background on these works, see *Muḥammed ibn Mūsā al-Khwārizmī: Le calcul indien (Algorismus)*, ed. André Allard (Namur, Belgium: Société des études classiques, 1992).

the Indian calculus widely throughout Western Europe.<sup>4</sup> It describes the methods, or algorithms, for the operations of addition, subtraction, duplication (that is, the doubling of a number), mediation (that is, the halving of a number), multiplication, division, and the extraction of both square and cube roots. To this the Old Norse *Algorismus* adds a section on a geometric progression between the cubes 8 and 27, connecting the sequence 8, 12, 18, 27 with the four elements of earth, water, air, and fire. The latter is an echo of a theme which dates back to at least the *Timaeus* of Plato. Although widely discussed in neoplatonic writings during late antiquity, as well as during the revival of platonism in the High Middle Ages, the connection between this geometric progression and the four elements has not been found in any other known versions of the Indian calculus.<sup>5</sup>

Numerous scholars have analyzed the text of the Old Norse *Algorismus* and, in particular, explicated the algorithms which it presents. Although similar to our modern methods for arithmetic, the presentation of these algorithms in the *Algorismus*, typically given without examples, is often obscure. In recent years, Kristín Bjarnadóttir and Bjarni V. Halldórsson have written extensively on the text itself and comparisons between the manuscripts in which it appears,<sup>6</sup> Otto Bekken has written on the historical and educational value of studying old arithmetical texts such as the *Algorismus*,<sup>7</sup>

- 4 Suzan Rose Benedict, *A Comparative Study of the Early Treatises Introducing into Europe the Hindu Art of Reckoning* (Concord: The Rumford Press, 1914), 12; André Allard, "The Influence of Arabic Mathematics in the Medieval West," *Encyclopedia of the History of Arabic Science*, ed. Roshdi Rashed, vol. 2 (London: Taylor & Francis, 1996), 523–24.
- 5 See, for example, William of Conches, *Guillelmi de Conchis: Glosae Super Platonem*, ed. Édouard A. Jeaneau (Turnhout: Brepols, 2006), 110–11. For a discussion of the sequence itself in the context of the Indian calculus, without reference to the four elements, see, for example, *Petri Philomeni de Dacia in Algorismum Vulgarem Johannis de Sacrobosco Commentarius*, ed. Maximilianus Curtze (Copenhagen: A. F. Høst & Fil. Bibliop. Reg., 1897), 74–75. For a more recent edition, see Petrus de Dacia, *Petri Philomenae de Dacia et Petri de S. Audomaro Opera quadrivialia*, ed. Fridericus Saaby Pedersen (Copenhagen: Societas Linguae & Litterarum Danicarum, 1983).
- 6 See, for example, Kristín Bjarnadóttir and Bjarni V. Halldórsson, "Ritgerðin Algorismus – samanburður handrita," *Vísindavefur: Ritgerðasafn til heiðurs Þorsteini Vilhjálmssyni sjötugum 27. september 2010* (Reykjavík: Hið íslenska bókmenntafélag, 2010) and "Algorismus: Hindu-Arabic Arithmetic."
- 7 See, for example, Otto B. Bekken, "Algorismus of 'Hauksbók': An Old Norse Text of 1310 on Hindu-Arabic Numeration and Calculation" (Agder: Agder distriktshøgskole, 1986).

and Abdelmalek Bouzari has detailed the origins of the Indian calculus and its path from Baghdad to Iceland.<sup>8</sup>

## The Manuscripts

A complete copy of the *Algorismus* appears in three manuscripts: in addition to the copies in GKS 1812 4to, folios 13v–16v, and AM 544 4to, folios 90r–93r, there is a copy in AM 685 d 4to, folios 24v–29r. A fragment of the text appears in AM 736 III 4to, folios 4r–4v, as well. AM 544 may have been copied between 1306 and 1308.<sup>9</sup> GKS 1812 is a composite manuscript. Written over a length of time stretching from the twelfth to the fourteenth centuries, it consists primarily of computational and related texts, along with maps, diagrams, and illustrations.<sup>10</sup> It is traditionally divided into four sections, each identified with a different scribe. The section which contains the *Algorismus* has been dated to the latter part of the first half of the fourteenth century and so is not much younger than the Hauksbók copy.<sup>11</sup> The other two copies are significantly younger: AM 685 d is most likely from the second half of the fifteenth century,<sup>12</sup> and AM 736 III is thought to be from the middle of the sixteenth century.<sup>13</sup> Kristín Bjarnadóttir and Bjarni V. Halldórsson have argued that the copies of the *Algorismus* in AM 544 and GKS 1812 are only once or twice removed from the same original, while the copies in AM 685 and AM 736 are drawn in part from the same stem but are significantly further from the original.<sup>14</sup>

- 8 Abdelmalek Bouzari, “The Calculus of Al-Khwārizmī,” *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*.
- 9 Gunnar Harðarson and Stefán Karlsson, “Hauksbók,” *Medieval Scandinavia: An Encyclopedia*, eds. Phillip Pulsiano and Kirsten Wolf (New York: Routledge, 2016), 271–72.
- 10 Gunnar Harðarson, “Medieval Encyclopedic Literature and Icelandic Manuscripts,” *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, 27–29.
- 11 Haraldur Bernharðsson, “Scribes and Scribal Practice in GKS 1812 4to,” *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, 63.
- 12 “AM 685 d 4to,” *ONP: Ordbog over det norrøne prosasprog* (Copenhagen: Den Arnamagnæanske Kommission), <https://onp.ku.dk/onp/onp.php?m135>.
- 13 “AM 736 III 4to,” *ONP: Ordbog over det norrøne prosasprog* (Copenhagen: Den Arnamagnæanske Kommission), <https://onp.ku.dk/onp/onp.php?m136>.
- 14 Kristín Bjarnadóttir and Bjarni V. Halldórsson, “The Norse Treatise *Algorismus*,” 75, and “*Algorismus*: Hindu-Arabic Arithmetic in GKS 1812 4to,” 189.

The Old Norse translation of the *Algorismus* was edited by P.A. Munch<sup>15</sup> in 1848 and Finnur Jónsson and Eiríkur Jónsson<sup>16</sup> in 1892–96. Both of these editions were based on the copy in Hauksbók, which has subsequently formed the basis for further study of the text. This copy is attributed to an Icelandic scribe, often referred to as “the first Icelandic secretary,” in the employ of Haukr Erlendsson.<sup>17</sup> The *Algorismus* appears in the manuscript between two sagas, with *Fóstbræðra saga* preceding it and *Eiríks saga rauða* following.

In contrast to the context of the copy in AM 544, the *Algorismus* in GKS 1812 appears immediately after a diagram illustrating some inscribed geometrical figures and precedes a short passage describing the division of a Latin unit of measurement, the *as*, into fractional parts. The leaves of the manuscript measure 210 mm by 140 mm.<sup>18</sup> Folios 13v through 16v have between thirty-two and thirty-five lines per page. The copy is attributed to a scribe who was most likely either Norwegian or at least trained in Norway, although he often uses Icelandic spellings, such as *blutum*, *oiofnn*, *henni*, *hin*, and *hinum* (rather than the Norwegian forms *lutum*, *uiofn*,<sup>19</sup> *henne*,<sup>20</sup> *in*, and *inum*), and writes in a cursive style compatible with Icelandic script of the time. Haraldur Bernharðsson conjectures that this scribe “may have been trained in the scribal milieu associated with the royal chancery and St. Mary’s Church in Oslo in the first half of the fourteenth century.”<sup>21</sup> There are a number of arguments to support this claim. For example, Haraldur points to the scribe’s use of the ligature “œ” for “æ” as typical for a scribe working in that setting.<sup>22</sup> Moreover, the consistent distinction between

15 P. A. Munch, “Algorismus, eller Anviisning til at kjende og anvende de saakaldte arabiske Tal, efter Hr. Hauk Erlendssons Codex,” *Annaler for nordisk oldkyndighed og historie* (1848): 353–75.

16 *Hauksbók*, eds. Finnur Jónsson and Eiríkur Jónsson (København: Thieles Bogtrykkeri, 1892–96).

17 “AM 544 4to: Hauksbók; Iceland and Norway, 1290–1360,” *handrit.is* (National/University Library of Iceland), <https://handrit.is/en/manuscript/view/en/AM04-0544>; Otto B. Bekken and Marit Christoffersen, “Algorismus,” *Medieval Scandinavia: An Encyclopedia*, 8.

18 “GKS 1812 4to, 13v–16v,” *handrit.is* (National / University Library of Iceland), <https://handrit.is/en/manuscript/view/is/GKSo4-1812>.

19 Although he uses this form once, in 14r/27.

20 Although he uses this form once, in 14r/31.

21 Haraldur Bernharðsson, “Scribes and Scribal Practice in GKS 1812 4to,” 112.

22 Op.cit., 110. Here Haraldur cites Eivind Vågslid [=Vågslid], *Norske logmannsbrev frå millomaldaren: Ei skrifthistorisk etterrøking av logmannsbrev frå Oslo, Uppland, Skien, Tunsberg*,

the vowels “œ” and “ø” makes it highly unlikely that it was written by an Icelandic scribe of this time.<sup>23</sup> Additionally, the scribe’s use of “r” with a superscript tittle above it to denote “ir”, as in “fingr” for “fingir” (that is, *fingr*), reflects “the regular notation in documents written in the period 1320–30 at the royal chancery in Oslo and the closely associated St. Mary’s Church.”<sup>24</sup> The use of the “œ” ligature is seen in such words as *næst* and *bæði*, written as *nœft* and *bœðe* in GKS 1812 but *neft* and *bæði* in Hauksbók. Other Norwegian influences are seen in words such as *hleypr*, written *løypir* in GKS 1812 but *hleypr* in Hauksbók, as well as words in which *g* is written *gh*, such as *merkingh* in GKS 1812 for *merking* in Hauksbók. However, the Norwegian influence gives us no indication as to where the text was composed. Moreover, the scribe who copied the *Algorismus* collaborated with the scribe who copied another section of the manuscript, and the evidence points to the latter scribe having been Icelandic.<sup>25</sup>

## The Transcription

The following transcription of the *Algorismus* from GKS 1812 4to is based on the black-and-white images at the Institut for Nordiske Studier og Sprogvidenskab in Copenhagen which were taken in 1982.<sup>26</sup> Color images are now available at *handrit.is*, a digital library at Landsbókasafn Íslands – Háskólabóksafn in Reykjavík; however, all but the last two of these images

*Borgarting og Bohuslän* (Oslo: Det Norske Videnskaps-Akademi i Oslo, 1930), 39, and Didrik Arup Seip, *Palæografi: B Norge og Island*, Nordisk kultur 28B (Stockholm: Albert Bonniers forlag, 1954), 121.

23 Op.cit., 110–12.

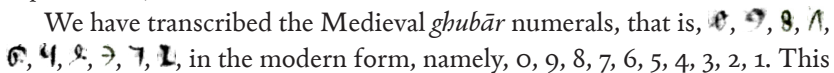
24 Op.cit., 112. Here Haraldur cites Didrik Arup Seip, *Norsk språkhistorie til omkring 1370*, 2. utgave (Oslo: Aschehoug, 1955), 137–39, and Jan Ragnar Hagland, *Riksstyring og språknorm: Spørsmålet om kongens kanselli i norsk språkhistorie på 1200- og første halvdel av 1300-talet* (Dragvoll, 1984), 161–62.

25 For more detail on the history of GKS 1812 4to and medieval Iceland encyclopedic works, see Gunnar Harðarson, “Medieval Encyclopedic Literature and Icelandic Manuscripts.” For more on the scribes of GKS 1812 4to, see Haraldur Bernardsson, “Scribes and Scribal Practice in GKS 1812 4to.”

26 “GKS 1812 4to, 13v–16v,” *digitalesamlinger.hum.ku.dk* (Institut for Nordiske Studier og Sprogvidenskab), <http://digitalesamlinger.hum.ku.dk/Home/Samlingerne/33608>.

have significant areas where the background is very dark, making it very difficult, and often impossible, to read the text itself.<sup>27</sup>

The transcription follows the guidelines of Menota at the diplomatic level.<sup>28</sup> As such, the transcript keeps the original punctuation, expands abbreviations, indicates certain types of errors, and distinguishes between the forms of letters only when they might have a phonological difference. In particular, a scribal addition above the line is denoted ``text``, a scribal addition in the margin is denoted `,text,`, and a scribal correction is denoted `|text|`. A dittography is denoted `|text|` and an addition by the editor is denoted `<text>`. Text that is illegible but is identifiable from the context is denoted `[text]`. The letters “f” and “þ”, “t” and “τ”, and “r” and “z” are not distinguished. The letters “d” and “ð” are not distinguished but are distinguished from “ð” although it is frequently difficult to ascertain the difference between the latter two. The letters “u” and “v” are distinguished although the scribe appears to use them interchangeably. Consonant ligatures for “pp” and “kk” are expanded silently. The letters “f” and “s” are distinguished since a positional difference can be observed in their use. Typically, “f” is seen in frontal and medial positions and “s” in final positions, but there are exceptions. For examples, we see a final “f” in *uerpilf* in 13v/27 and 15v/18 and 20, and *liof* in 16v/6, while *subdupli* in 15r/33 and *sem* in 16v/6 both have a frontal “s”. Additionally, an initial capital “s” is written “S”, as in *Setta* in 13v/24 and *Seaunda* in 13v/25. Abbreviation symbols are expanded with italics, suspensions are expanded in parentheses, small capitals are not expanded, and all accents and punctuation are shown. For this transcript, we have expanded the “er/ir” abbreviation with “ir”. So, for example, “fior” is expanded as “fiorir” and “fingr” is expanded as “fingir”. We have indicated folio and line numbering within the text and have divided the text into sections as indicated by the scribe (either by a drawing of a hand pointing to the right, blank space, or the use of a large capital letter).

We have transcribed the Medieval *ghubār* numerals, that is, , in the modern form, namely, 0, 9, 8, 7, 6, 5, 4, 3, 2, 1. This

27 “GKS 1812 4to, 13v–16v,” *handrit.is*. The enhanced readability of the black-and-white photographs may indicate that the photographer used ultraviolet illumination, although no special technology is noted in the registration.

28 “Menota Handbook 3.0,” *Menota Nordic Text Archive* (Menota, 2019), [https://www.menota.org/HB3\\_ch4.xml](https://www.menota.org/HB3_ch4.xml).


transcription is consistent with the diplomatic edition principle of ignoring orthographic differences that have no consequences in interpreting the text. Yet it has the disadvantage of making the text look more familiar to us than it really is.

## The Translation

The translation tries to be as literal as possible while still being rendered in smooth English. The line between “literal” and “smooth” is not a clear one, yet it is clear that a literal word-for-word translation would make the text more opaque to an English reader than it really is, while an overly polished translation would make the text read much like a modern primer in arithmetic.<sup>29</sup>

The text refers to each of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 as a *stafr*, which we have translated as “character.” As numbers, each of 2 through 9 is referred to either as a *fingr* or a *figura*, which we render as “digit.” A multiple of 10 is called a *liðr*, which we render here as “article.” The words *fingr* and *liðr* are literally “finger” and “joint,” and are themselves literal translations of the Latin *digitus* and *articulus*. These words are indicative of the ancient form of calculating with fingers.<sup>30</sup>

In some places we have translated *leiða* as “to multiply,” which, although not literal, is indicated by the context and seems to be a direct translation of *ducere* in the Latin versions. The text refers to subtraction in two ways, initially as *afdráttir*, a literal translation of the Latin *subtrahere* (“to draw off”), but most frequently as *taka af*. We have translated the latter as “take away”; although it is a less formal way to say “subtract” in English, it is literally correct and maintains the distinction between *taka af* and *afdráttir*. Moreover, it is analogous to the use of the Latin *demere* (“to take away” or “subtract”) in the *Carmen de Algorismo*.

The *Algorismus* calls the symbol for nothing, , a *cifra*. The various versions of the Indian calculus refer to this symbol in numerous ways. For example, in his commentary on the *Algorisumus Vulgaris* of John

29 The algorithms for performing the operations of arithmetic in the *Algorismus* are, except for some details of implementation, essentially those still taught in primary schools today.

30 See, for example, Menso Folkerts, “Early Texts on Hindu-Arabic Calculation.” *Science in Context* 14, nos. 1-2 (2002): 13.

of Sacrobosco (c. 1165–c. 1256), Peter of Dacia (c. 1250–c. 1310) wrote: “Decima vero o dicitur teca, circulus, vel cyfra, vel figura nichili, quoniam nichil significat.”<sup>31</sup> [But the tenth, o, is called *teca*, circle, or cipher, or figure of nothing, because it signifies nothing.] Peter supposes the word *teca*, or *theca*, to be derived from the name of an iron used to brand thieves, but it may simply come from its resemblance to the Greek letter  $\theta$ .<sup>32</sup>

We are aware of several translations of the Old Norse *Algorismus* into modern European languages. Munch provided a translation into Danish along with his edition;<sup>33</sup> Otto Bekken and Marit Nielsen translated the text into Norwegian;<sup>34</sup> and Kristín Bjarnadóttir has published a version in modern Icelandic orthography.<sup>35</sup> All of these were based on the copy in Hauksbók. We are not aware of any previous translations into English of any versions.

## Conclusion

The intent of this paper is two-fold: first, to provide a transcript of the second of two early witnesses to the Old Norse *Algorismus* and, second, to present an English translation. Although normalized editions of the versions of the *Algorismus* in Hauksbók and GKS 1812 would differ in only a few places, a diplomatic transcript of the latter version is of historical interest in underscoring the distinctiveness of the text and in illustrating the variance in fourteenth-century Icelandic script. In addition, the English translation will make this important work more easily accessible to the international research community.

31 *Petri Philomeni de Dacia in Algorismum Vulgarem Johannis de Sacrobosco Commentarius*, 2.

32 David Eugene Smith and Louis Charles Karpinski. *The Hindu Arabic Numerals* (Boston: Ginn & Company, 1911), 61.

33 Munch, “Algorismus, eller Anviisning til at kjende og anvende de saakaldte arabiske Tal, efter Hr. Hauk Erlendssons Codex.”

34 Otto B. Bekken, Marit A. Nielsen, and Steinar Thorvaldsen, *Algorismus i Hauksbok: En norrøn regnetekst fra 1300-tallet* (Tromsø: Eureka Digital, 2010).

35 Kristín Bjarnadóttir, “Algorismus: Fornt stærðfræðirit í íslenskum handritum,” *NETLA* (2004), <http://wayback.vefsafn.is/wayback/20201017180549/https://netla.hi.is/grein-ar/2004/001/index.htm>.



*Algorismus* in GKS 1812 4to, folios 13v–16v  
Diplomatic Edition

[13v] Lift þessi heitir algorifmus. hana fundo fyft induerkir menn með<sup>36</sup> tíu stofum |<sup>2</sup> er sua eru ritadir ·0 ·9 ·8 ·7 ·6 ·5 ·4 ·3 ·2 ·1· hín fyfti stafuір merkir |<sup>3</sup> ein j fyfta stað. En annar tua. En þriði. þríá. Ok huer efttir þui sem |<sup>4</sup> fkipadir er alt til hins síðarfta<sup>37</sup> er cifra heitir. ok skaltu þessa stafuір fra |<sup>5</sup> hōgre hende upp hefuia ok rita till vinftri handar sem ebreifku. |<sup>6</sup>

h. Hver þessi stafuір merkir sik einfalligha í fyrfta stað En ef han er j |<sup>7</sup> aðrum stað þa merkir han .x. finnum sealfuan sik. Ok j huern stað er þu |<sup>8</sup> fetir nokkora þessa figuru. þa merkir hon avalt tíu hlutum meira j þeim |<sup>9</sup> stað er till uinftri handar veit. heldir en í noofta stað adr. Cifra merkir |<sup>10</sup> ekki fir sik en hon gerir stað. ok gefuір adrum figurum merkingh |<sup>11</sup>

þ. ar nooft höyrir þat till at vita þrenna grein stafuanna ok allrar to|<sup>12</sup>lu. þui at aul tala mínne en tíu. heitir fingir. En su tala oll er tegum |<sup>13</sup> gengnir. heitir liðir. huart sem hon er meiri eðir mínni. En su tala er alt |<sup>14</sup> er saman liðir ok fingir. heitir samfett tala. |<sup>15</sup>

E. f þu uilt rita nokkora tolu. þa hyg. þu at. ef þat er fingir. ok rita |<sup>16</sup> j fyfta stað eina hueria figuru flika sem þarf a þessa leið .8. En ef þu uilt |<sup>17</sup> lid. rita. þa settu cifru firir figuru. a þessa lund .70. Vilt þu samfetta |<sup>18</sup> tolu rita. þa settu figuru<sup>38</sup> firir lid. sem her .65. |<sup>19</sup>

h. ueria tolu er þu ritar þa er hon íofnn ef tígum gengnir. eðir jafnn fingir er umfram |<sup>20</sup>

En oll tala er oiofnn. ef oiafnn fingir er umfram Jafnnir fingir eru fiorir. |<sup>21</sup> 2.4.6.8. En oiafnnir. aðrir fiorir .3.5.7.9. En ein<sup>39</sup> er huarki þui at |<sup>22</sup> han er eigi tala heldir uphaf<sup>40</sup> allrar tolu.<sup>41</sup>

J. seau staði er fkipt |<sup>23</sup> greinum þessar listar. Heitir hin fyfta uidir laghningh. Annur afrattir |<sup>24</sup> Þriðia tuefaldan Fiorda helmínga fkiptí. Fimta margfaldan Setta |<sup>25</sup> fkiptíng. Seaunda at taka rot vndan. oc er su

36 13v/1 með] *Hauksbók* has ok skipvdr med.

37 13v/4 síðarfta] there is an “ir” abbreviation mark over the “r”.


38 13v/18 figuru] *Hauksbók* has fingr (digit), which is correct.


39 13v/21 ein] possibly ein; *Hauksbók* has einn.

40 13v/22 uphaf] the “h” appears to have an abbreviation symbol.

41 13v/22 tolu] white space may have been left for the insertion of a hand pointing to the right to indicate a new section, as seen in line 30 below. In contrast, *Hauksbók* has section titles.

með tueimír greinum<sup>26</sup> Annur er at taka rot vndan firir íkóyttri tolu. En onnur greín er<sup>27</sup> þat at dragha rot undan atthyrndrí tolu þeirri er uerpilf uoxst hefuír<sup>28</sup>

Þra enní hògre hendi íkalt þu af taka ok vidlegía ok íkipta j helmin<sup>29</sup>ga. En fra uinftri hendi skal þu tuefalda ok íkipta ok margfalda<sup>30</sup> Oc fva dragha vndan rot huaratueggio. 

Ef þu uilt adra tolu<sup>31</sup> uid adra leggía. þa rita ifuír uppi ena meiri toluna. oc fet ena<sup>32</sup> mínne tolu iamfram till ennar hògre handar. Ok leg þa Fíguu [14r] fyrft up uid toluna er utarf er til hògre handar oc ef fu tala oll faman<sup>3</sup> er fingir. þa rita han j fama ftað. En ef talan uerdír famsett þa rita<sup>3</sup> j fyrfta ftað fingir. En legh uidír lidin þa toluna fem j noofta ftað er adír<sup>4</sup> En ef lidír uerdír af uidlagning. j fyrfta ftað þa rita þar cifru: En legh<sup>5</sup> lidin uid þa tolu er nooft fteudír ef þar er nokkur tala ella rita han þar<sup>6</sup> ein faman. En ef þar er cifra þa tak hana brot. En fet lidin þar niðír<sup>7</sup> Legh fidan aðrar figurur uid at flikum hootte. 

Ef þu uilt adra tolu<sup>8</sup> af annarre taka. þa rita tuennar tolur. fem i uidírlagningh. ok fet íam<sup>9</sup>nan ena minne tolu undír. ellighar iamfna. þa tak þu af enne fyrftu<sup>10</sup> figuru þa tolu fem vndír fteudír ef þat ma. oc rita ef nokkot er efttír j fama<sup>11</sup> ftað. ella fet þar cifru. En ef þu mat eighi ena fyrftu figuru af taka<sup>12</sup> oc er fu tala meiri er undír fteudír. þa tak þu ein af nooftu figuru. oc gae<sup>13</sup> þerff. at hon geri tio j fyfta ftaðenum. Tak þa af þeim alla tolu þa fem<sup>14</sup> undír er. ok i fama ftað þat fem af löypír. En ef cifrur ftaða uppi ifuír<sup>15</sup> þa tak ein af þeirri figuru er nooft fteudír cifrum ok rita nio. Þar fem cifrum<sup>42</sup>. <sup>16</sup> {ftoðo} `voru'. alt þar till er þu kemír j þan ftað er þu uilt af taka. Ok tak þu<sup>17</sup> af þeim tiu fem þarf. ok rita j fama ftað þat er lifnarr.

Ef þu uilt<sup>18</sup> tuefalða nokkora tolu. þa rita fyrft flika tolu fem þeer likar. Þar nooft<sup>19</sup> tuefalda þu. þa figuru er meft ueit till uinftri handar ok rita j noofta ftað<sup>20</sup> þat er af löypír sua fem j uidírlagningh en ef femis fteudír ifuír uppi<sup>21</sup> j yðfta ftað þa legh uid ein. Þui at þar uar adír íomfn<sup>43</sup> tala er j helminga uar íkift<sup>44</sup><sup>22</sup>

42 [14r/15 cifrum] context indicates this should be cifrur.

43 [14r/21 íomfn] should be óiofn. This is clearly an error, which appears as well in Hauksbók (but not in AM 685 d 4to). The text is referring to the case in which the given number was the result of halving a previous number, resulting in a remainder of one-half. Hence the original number must have been odd.

44 [14r/21 íkift] added below the line.


En ef þu uilt helmíng h af taka rita flika tolu fem þu uilt <sup>|23</sup>ok tak af helmíng h enni fyftu *figuru* ef hon er iofn. En ef hon uar oiofn þa <sup>|24</sup>skípt j helmíng h þui er af einum löypír ok tak vp ein en rita ifuír uppi þan <sup>|25</sup>ftaf er helmíng h<sup>45</sup> huerslutar merkir ok uer kollum femís ok sua er gior .¶. en set <sup>|26</sup>cifru j ftadin.<sup>46</sup> þar nooft tak af annarre *figuru* helmíng h at fama hootte ef <sup>|27</sup>hon er iofn. En ef hon er uiofn. þa tak af helmíng h af þui er iamt er. oc <sup>|28</sup>af up þan ok ger af honum fim j noofta ftad þui at þat er helmíng h af <sup>|29</sup>tío En ef j adrum ftad ftendir ein þa tak han up ok rita fim j noofta <sup>|30</sup>ftad. en set þar cifru. fem han stoð. Ekki gerir cifra nema nokkur <sup>|31</sup>figura ftande till uinftri handar henne. Far fidan fram at flikum hootte <sup>|32</sup>huerffu margar fem *Figurur* ero.

Ef þer likar at margfalda [14v] ,adra, toluna j adra. rita tuennar raðir ftafuanna með þeim hootti at hín <sup>|2</sup>yzfta *figura* þeirrar tolu er þu marghfaldar ftandí undír fyfta <sup>|3</sup>ftaf ennar ðfre tolu. en til vinftri handar allar aðrar fra þeirri þoor <sup>|4</sup>fem undír eru. þar nooft skalt þu hugfa huerffu mikít ena meiri *figuru* <sup>|5</sup>skortír a tíu. þa er þu uilt marghfalda. Ok sua marghar einngar <sup>|6</sup>fem askortír a tíu. sua opt skalt þu ena minní toluna þaa er þu <sup>|7</sup>uilt margfaldda taka af tíghum hennar. ok at þu skilir þetta marg<sup>|8</sup>faldda seu ok níu . Nío skortír ein a tíu. þui tak þu eína seu af <sup>|9</sup>seautíghum. þa uerda efttír þrír ok sextíghi. þat eru seu finnum <sup>|10</sup>níu. At flíku íkapi mat þu aðrar tolur róyna. Margfalda hína <sup>|11</sup>fyftu *Figuru* retligha j allar þoor er undír standa. ok rita ifuír huerri <sup>|12</sup>*Figuru* þa margfaldan er hon hefuír oc till uinftrí ,handar, þat fem eigi ma <sup>|13</sup>ifuír henni (standa) í noofta ftad með uidlaghning rettrí. ok þa er þessi *figura* <sup>|14</sup>er margfoldut foor ena yzftu af þeim fem undír ftanda. Vndír nooftu <sup>|15</sup>*figuru* ok margfaldda uid þan sua fem vid enn fyrra: ok ef margfalddan gefuír <sup>|16</sup>þer líd set cifru ifuír uppi ok skípa lídnum till vinftri handar. En ef <sup>|17</sup>boode uerðír af. margfalddir fíngír ok líðir. þa rita fíngír ifuír þeirri |talur| *Figuru* <sup>|18</sup>er þu margfalddadír. en líd j ennoofta ftad. En ef fíngír ein uerdír af mar<sup>|19</sup>gf(aldan) þa rita hæn ifuír uppi. Ef cifra er i enne ðfre tolu þa laup ifuír hana <sup>|20</sup>þui at ekkí er hennar margfalddan. Þerff skal en ok gá. at taka af <sup>|21</sup>*figurur* þoor fem uppi eru ifuír settar íamskíot huería fem þu. hefuír marg<sup>|22</sup>faldat. ok rita þan fíngír j ftad huerrar fem till hóyrír edir cifru <sup>|23</sup>ef þat er rettarra. En legh þat uid

45 14r/25 helmíng h] should be helmíng h (helmíng in *Hauksbók*).

46 14r/26 ftadin] is missing context. Here, and in *Hauksbók*, some text is missing. The instruction to set cifru j ftadin evidently refers to the case when a one is in the first place.

hínar er till vínfra uegs |<sup>24</sup>ftanda sem af löypir. ef cifra ftendir ifuir þeirri figuru er þu margfalðar |<sup>25</sup>þa tak hana af. ef fingir uerdir. oc margfallda.<sup>47</sup> ella stanðe hon kyr |<sup>26</sup>

Ef þu grunar huart þu hefuir ret margfaldað. þa skipt j |<sup>27</sup>fundir alla toluna vm margfalðan. þat er fu tala er undir ftod. oc |<sup>28</sup>mant þu fa hina somu tolu oc fyrft hafðir þu. 

Ef þu uilt |<sup>29</sup>skípta j fundir tolunne þa rita tuernnar raðir stafanna ok rita vndir |<sup>30</sup>ena mínne toluna. ok skal en meíri tala uera halfu meíri eðir þriu |<sup>31</sup>flik. eðir meíri munir.

Set þu ena Fremrri<sup>48</sup> figuru þa er undir ftendir |<sup>32</sup>gengt enne fyrftu. ifuir uppi. ok aðrar til högre handar iamfram |<sup>33</sup>sem þ`oo`r endast er undir ftanda. þar nooft hugfa þu vm huerffu opt hin |<sup>34</sup>fysti fingir er i oc einum ófra. fua at íafn opt se þoor er fylgía [15r] henni huer j þeirri tolu er ifuir ftendir ok fet þu þan fingir gengt enne |<sup>2</sup>yzftu figuru er undr ftendir ok þo uppi ifuir baðar raðir . Tak sidan ena |<sup>3</sup>fyrftu af enni fyrftú figuru ok þar nooft hueria at hendi íafn opt af enne |<sup>4</sup>ófrí taulu. En ef ein tala er undir þa tak hana af enne ófre |<sup>5</sup>enn|n| |<sup>5</sup>tolunne. Þar nooft flyt alla tolu þa er undir ftendir vm ein ok fin |<sup>6</sup>annan quociens ok fet þan hía hinum fyfta ok tak hína neðre tolu fua |<sup>7</sup>opt af enne ófre ok ger at fama hootte fua opt sem þarf. Ef þu |<sup>8</sup>mat eígi ena |<sup>8</sup>neðri| tolu eðir figuru finna j enne ófre þa fet þan |<sup>9</sup>fingir er undir ftendir fremstir nooft enni fyrftu ok aðrar at fama |<sup>10</sup>hootti till högre handar ok fin sidan quociens eptir flikum hootte ok |<sup>11</sup>fór afttir figurur sem þarf ok rita alla fama quociens ifuir uppi fua marga |<sup>12</sup>sem þarf. En ef cifra ftendir niðri undir þa laup ifuir hana. þui at |<sup>13</sup>ekki ma henni skipta. Þa er þu kemir vndir ena yðftu figuru. oc |<sup>14</sup>hefuir hinnne<sup>49</sup> skipt mat þu ekki lengir skipta. oc goot þu þa þeirrar |<sup>15</sup>tolu er eftir ftendir ef hon er nokkur. Ef þu uilt profua huart þu |<sup>16</sup>skiptir ret þa margfalda þa tolu er undir ftod uid quociens. Oc |<sup>17</sup>mant þu fa somu tolu ok fyft hafðir þu. En ef nokkot líop fram af j |<sup>18</sup>skíptingh þa leg þat uid sidan er margfaldað er ok mant þu finna hína somu tolu. |<sup>19</sup>

Þa er þu leiðir eína huería tolu ok margfalddar í fealfua |<sup>20</sup>fik. heitir fu tala ferlkóyt eðir quadrans. oc en fyfta tala fu er |<sup>21</sup>þu margfalðar heitir rot.

47 14v/25 oc margfallda] possibly should be af margfalldan (*Hauksbók has oc margfallda as well*).

48 14v/31 Fremrri] apparently corrected from meíri.

49 15r/14 hinnne] *Hauksbók has henni*.

oc er huer tala rot undir nokkoro`ri' tolu |<sup>22</sup> En eigi er huer tala ferfkóyt. Ef þu uilt rot finna undir nokkor|<sup>23</sup>ri tolu. þa rita fyft flika tolu er þer likar. ok i enum fyfta viofn|<sup>24</sup>num stad rita undir fingir þan er þu leidir j sealfuan fik. ok taki |<sup>25</sup> af þat er ifuir uppi er eðir sua fem nooft ma han ganga. Siðan tuefal|<sup>26</sup>da þu þan fama fingir. oc heitir þat dupl. tak þa up fingrin. oc |<sup>27</sup> heitir han subdupl. goot þu subdupls. en rita dupl í noofta stað ef þat |<sup>28</sup> er fingir. en ef líðir er þa rita þar fem fingrin fyrri stöð ok set cí|<sup>29</sup>fru firir. ella fingir. ef samset tala. Fin siðan nyian fingir oc <leið> han |<sup>30</sup> j ðupl. oc tak af enne ófre tolu þa tolu er þu margfaldadír. Sidan |<sup>31</sup> margfalda þu fingir j sealfuan fik ok tak þa tolu af enne ófre |<sup>32</sup> gengt sealfuum honum. þar nooft tuefalda þu fingrin ok goot hans med |<sup>33</sup> fyrra subdupli. ok set dupl j noofta stað sem fyr. Fin þar nooft |<sup>34</sup> nyian fingir ok leid j duplín booðe samt. ok flytir<sup>50</sup> duplet fyrri [15v] at hinu duplí um ein stað. ok leg þar uid ef þar stöð lidir firir |<sup>2</sup> af hinu duplinu. Margfalða þa nyian fingir j booðe ðuplin. ok |<sup>3</sup> tak þa tolu af enne ófre gengt duplino. Ger at fama hootte |<sup>4</sup> sua opt fem þarf. ok leid nyian fingrin j aul duplín. ok flyt þau |<sup>5</sup> eftir aualt vm ein. þar til er þu kemir j enn ydsta stad. Ef upp |<sup>6</sup> gengir oll su talan er þu ritadir i fyrstumme. þa uar su tala |<sup>7</sup> ferfkóyt. En rot vndir þeirri tolu eru fingir allir saman þeir |<sup>8</sup> er þu tuefaldaðir. með sidarfta fíngrinum þeim er þu fant. |<sup>9</sup> Margfalda þu rotina j sealfua fik ok mant þu hafua hina |<sup>10</sup> fumu<sup>51</sup> tolu fem j fyftu ef þu gerdír ret. Ef af löypir tolunne |<sup>11</sup> nokkot þa er þu dreghr rotina undan. þa uar su tala eigi fer |<sup>12</sup> íkóyt ok leg þu þa tolu uid hina er þu margfaldadír rotina till |<sup>13</sup> oc man þu fa ena fyftu toluna ok er su tala ol saman rotín oc |<sup>14</sup> af laup rot meiri tolu. En ef fyfti stadir þeirrar tolu er þu |<sup>15</sup> ritadir uar iaft þa fin fingir undir nooftu figuru ok margfal|<sup>16</sup>ða a somu leid. ☞

Ef þu margfalðar retligha þa leid fer |<sup>17</sup> íkóytta tolu j sealfua fik. ok su tala er af þeirri margfaldan cemir |<sup>18</sup> heitir cubicus edir uerpilf tala. hun er alla uega íam mikil. En |<sup>19</sup> rotin undir cubico uar en fama. ok ferfkóyttrar tolu. huer |<sup>20</sup> rot er<sup>52</sup> nokkorrar uerpilf tolu edir cubici en eigi er huer tala |<sup>21</sup> cubicus.

50 15r/34 flytir] Hauksbók has flyt here, which is the correct imperative.

51 15v/10 fumu] Hauksbók has somu, that is, sqmu, here, as does GKS 1812 elsewhere.

52 15v/20 hver rot er] text is corrupt. The correct reading, namely, hver tala er rot, is found in Hauksbók. Here "b." is written above rot and "a." is written above er, indicating that the word order should be inverted.

Ef þu uilt finna <rot><sup>53</sup> vndir cubíco. hugfa huerffu mikil |<sup>22</sup> tala er. ok huerffu margir stadir ero. Fín þar nooft fingir |<sup>23</sup> j enum fremsta þufunda stað. Þufunda staði kollum uer |<sup>24</sup> þa alla er um þufundir éinar bríotast. þat er en fiordí |<sup>25</sup> oc en feau<n>dí ok en tiundi ok en þrettandi ok af afalt |<sup>26</sup> löypir ifuir tua stadi.

Fra uinftri hendi skal þu þetta |<sup>27</sup> uerk uphefuía. leid þan fingir er þu fant j fik cubice |<sup>28</sup> þat er tuisvar sinnum margfaldað fyrft j sealfan fik |<sup>29</sup> oc annat sín j þa tolu er þar kom af. ok þar nooft [ta]k af |<sup>30</sup> ófre tolu þessa tolu alla gengt *figrinum þeim* sealf[um]. Oc |<sup>31</sup> þrefalda þar nooft *figrín*. ok hoppa ifuir éin stad með þa |<sup>32</sup> tolu ok fet j þridía stað firir hínun þa toluna ef þat er |<sup>33</sup> fingir. En ef þat er lidir set þar cífri en lidín j noofta |<sup>34</sup> stad. En ef samfet tala er þa fet *figrín* j fama stad. |<sup>35</sup> en líd et noofta. Þar nooft fín nyian fingir j noofta stad [16r] þrefaldri tolu er tripl heitir ok leid han með hinnu *figuru* er fyft fant þu |<sup>2</sup> Ok uer kollum subtripl oc a högra uegh henni j triplit með margfal|<sup>3</sup>ðan. ok þar nooft leid han ein saman i þa tolu er af margfaldan kom |<sup>4</sup> oc uer kollum productum. Tak þa þessa tolu alla samt af enni ófri |<sup>5</sup> gengt þui er tripl stod. þui nooft leid fingir þan fama i sealfuan fik cu|<sup>6</sup>bice. oc tak þa tolu af enni ófri gengt sealfum *figrinum*. Tak þan |<sup>7</sup> fingir af ok þrefalda her fem *enn* fyrra ok fin þa nyian fingir. Leið |<sup>8</sup> han með baðum subtriplinn oc triplin samt. oc flyt afalt triplin |<sup>9</sup> fornari eptir fem þu gerir i minna rotardrat uið dupl. nema her |<sup>10</sup> skalt þu afalt ifuir ein stad hoppa en leggja þo at fama hœotte tripl |<sup>11</sup> uid tripl með rettri uidlagningh Far fram at sliku hofui meðan |<sup>12</sup> þarf oc þu kemir j yðsta stad. En þat skalt þu með mikilli uanðuircð |<sup>13</sup> huxa þa er þu finnr fingirna at þeir taki eigi sua mikit af ófri tolu |<sup>14</sup> at fu tala hafin<sup>54</sup> eigi stað er þu margfaldar triplin till. eðir hín |<sup>15</sup> onnur er þu margfaldar *figrin* till þan siðarra. Varðueit þu afalt |<sup>16</sup> subtripl með tripli. goot þess ok ef cifrur koma. j subtripl at engi er |<sup>17</sup> margfaldan eðir þrefaldan þeirra. en halda þoor stoðum *sinum* meðan nok|<sup>18</sup>kur *figura* er till högre handar þeim. Ok er þat vuanðast j vidirlaghning |<sup>19</sup> tripls. afalt fer þat sem fyr er rítat j vidrlogho líft.

Fingir allir |<sup>20</sup> samt þeir er subtripl voro ok yztir fingir með ero rot *enn*-ar meiri tolu |<sup>21</sup> þeirrar er þu ritaðir fyft ef up gek oll talan j af droottinum ok marg|<sup>22</sup>falda þu subtriplin j sealfuan fik cubice. ok mun þu finna hina fyrftu |<sup>23</sup> tolu. En ef aflíop nokkot tolunní j afdrootti. þa er fu tala eigi

53 15r/21 <rot>] *Hauksbók* has the same omission.

54 16r/v14 hafin] *Hauksbók* has hafi.

cubicus |<sup>24</sup> En þo er aflaup þat með fubtriplum rót nokkurs cubíci. Oc ef þu mar|<sup>25</sup>gfaldar rót ena mínni. cubice. oc leg uid þa tolu er af margfaldan |<sup>26</sup> kemir aflaupít. oc mant þu fa fyrstu tolu er þu rítaðir Oc nu rí|<sup>27</sup>tum uer at finne eigi fleira þar af.

Þessar eru fingra margfal|<sup>28</sup>danar ferfköyttrar. af. 3. 9. quadratus. af 2 4. Af .4. 16. |<sup>29</sup> quadratus af. 5 25 quadratus. Af .6. 36. quadratus. Af .7. .49. quadratus |<sup>30</sup> Af .8. 64. quadratus. Af .9. 81. quadratus. Oc er fu list till at finna fingra |<sup>31</sup> margfaldanir sem ritud er fyr. Þessi er fingra margfaldðan cubice |<sup>32</sup> 3 rot 27. cubus. 2 rot 8. cubus. 4. rot 64 cubus 5 rot .125. |<sup>33</sup> cubus .6. rot 216. cubus. 7. rot .343. cubus. 8. rot 512. cubus |<sup>34</sup> 9 rot 729 cubus.

Huer ferfköyt tala hefuir tuor mooling|<sup>35</sup>gar þat er breid ok lengð. En cubicus tala hefuir þrenna moolingh. þat [16v] er breid ok lengð ok þycð edir hood. Oc þui kalla spekingar huern synilighan |<sup>2</sup> likama með þessi tolu saman settan. at han hefuir. jamnan<sup>55</sup> þessa moolingh |<sup>3</sup> þrenna.

Með þui at elif<sup>56</sup> speki. oc ein gud uilde heimin synlighan ok |<sup>4</sup> likamlighan ikapa. þa fetti han fyrst tuor ennar yðstu hofuðskempn|<sup>5</sup>nur eld ok jord. þui at ekki ma naturuligha syniligh vera uttan þoor |<sup>6</sup> Þar sem elddir gerir liof ok roringh. En jord staðfesti ok hald. En |<sup>7</sup> með þui at þau hafua þrenna víamna huiligleika ok gangstadrígha |<sup>8</sup> þa. uar naturuligh naudsyn at setia nokkot milli þeirra þat er samþykktí |<sup>9</sup> þeirra vfootti. Ok sem fyr er saght at elddir ok jord ok þat alt sem likam|<sup>10</sup>light er er<sup>57</sup> með þrefaldre tolu er uer kollum cubicum saman fet |<sup>11</sup> þa ritum uer þessa tua cubus. Ritum uer jordina þessa leið. Tuifuar |<sup>12</sup> finnum tueir tuifuar. 248.

En eldin sua þryfuar þrir þryf|<sup>13</sup>uar. 3927.

En með þui at ekki eit midfkeid ma milli þessarra |<sup>14</sup> talna einna þat er jamre luttegningh hóyrir till huartueggia. |<sup>15</sup> ok engra annarra tueggia cuba. þa finnum uer tuor lutfellingar |<sup>16</sup> tolur a þessa lund. leidum rot enf meira cubs j quadratum ens |<sup>17</sup> mínna<sup>58</sup> cubs þat er tuyfua tueir þryfuar. 24. 12. Oc rot enf minna |<sup>18</sup> cubs j quadratum ens meira cubs þeim<sup>59</sup> er þryfuar þrir tuyfuar |<sup>19</sup> .39. 18. Þessar tuor tolur hóyra íafnt til tueggia hinna enu yðstu cu|<sup>20</sup>ba. þui at feau ok .xx. hafua j fer. 18. ok helmíng h af .18. En .18. hafua |<sup>21</sup> j fer .12. ok helmíng h af .12.

55 16v/2 jamnan] Hauksbók has avallt.

56 16v/3 elif] Hauksbók has the correct eilif.

57 16v/10 er er] Hauksbók has er. þa er.

58 16v/17 minna] appears to be corrected from meira.

59 16v/17 þeim] Hauksbók has þeim as well, but both Fimmur Jónsson and Munch correct to þat.

Sua hafua ok .12. j fer .8. ok helmíng<sup>|22</sup> af .8. at fama hootti fkalt þu afalt luttekníngar fírna millí<sup>|23</sup> tueggía cuba.

Sua skipadí gud tuemnar hofutlkepnur millí eldz<sup>|24</sup> ok íarðar. lopt ok vatn. oc hefuír uatn tua huilgleika af íorð oc<sup>|25</sup> tuoor tolur. En af elddí eín huilgleik ok eína tolu.

En lopt<sup>|26</sup> hefuír tua huilghleika af eldí ok tuoor tolur. En eín af íorð ok<sup>|27</sup> eína tolu. Oc er elddír þuí lettari en lopt fem .27. eru meí<sup>|28</sup>rí en .18.

En lopt þuí lettari en uatn fem .18. eru meír en .12. <sup>|29</sup>vatn þuí lettara en jorð fem .12. eru meír en .8.

Þetta <er> full<sup>|30</sup>lígat at skilía<sup>60</sup> j þeirri figuru er her er fíðar gor oc kollud er cubus<sup>|31</sup> PErFectus

### Translation of the *Algorismus* in GKS 1812 4to

[13v] This art is called *algorismus*. First discovered by men of India, they used ten characters, which are so written: 0 9 8 7 6 5 4 3 2 1. The first character denotes one in the first place, the second two, the third three, and each according to how it is placed until the last, which is called a *cipher*. You shall write these characters from right to left as in Hebrew.

Each character denotes itself simply in the first place. But if it is in the second place, then it denotes X times itself. And in each place that you place some figure, then it always denotes ten parts more with respect to that place which points to the left, relative to the next place before. The cipher denotes nothing in itself, but it creates a place and gives the previous figures signification.

Next it is appropriate to know a three-fold distinction of the characters and of every number. Every number which is less than ten is called a digit.<sup>61</sup> Every number which is made from groups of ten is called an article,<sup>62</sup> whether it is bigger or smaller. But a number that is both an article and a digit is called a compound number.<sup>63</sup>

If you want to write some number, then examine if it is a digit and

60 16v29–30 Þetta <er> fullígar at skilía] *Hauksbók* has Ma þ<et>ta fvllígar skilía.

61 Literally, “finger.”

62 Literally, “joint”; Latin “articulus.”

63 *Samsett tala*, literally “composite number,” is rendered as “compound number” to avoid confusion with the conventional definition of a composite number.



write in the first place any figure such as needed, for example, 8. But if you want to write an article, then put a cipher before the figure, like this: 70. If you want to write a compound number, then set the figure<sup>64</sup> before the article, as here: 65.

A number is even if it is made from groups of ten or if an even digit is in the front. A number is odd if the digit in front is odd. There are four even digits: 2, 4, 6, and 8. And the other four are odd digits: 3, 5, 7, and 9. But one is neither [even nor odd] because it is not a number but rather the origin of all number.<sup>65</sup>

There are seven branches of this art. The first branch is called addition, the second subtraction, the third doubling, the fourth dividing in half, the fifth multiplication, the sixth division, and the seventh to take a root. And there are two branches for this: One is to take a root of a squared number and another type is to extract a root from an octagonal number<sup>66</sup> which has the shape of a cube<sup>67</sup>.

From the right you should take away from, add, and divide in half. From the left you should double, divide, multiply, and also extract both types of root.

If you want to add one number to another, then write the larger number above and set the smaller number even to it on the right. Then first add the figure [14r] up to the number which is farthest out to the right. And if this entire number is a digit, then write it in the same place. But if the number is a compound, then write the digit in the first place and add the article to that number which is in the next place before. But if an article results from the addition, then write a cipher in the first place and add the article to that number which stands next if some number is there, or, else, write it there alone. But if there is a cipher there, then remove it and set the article down there. Then add the other figures in the same way.

If you want to take one number away from another number, then write the two numbers as in addition and always set the smaller number below, otherwise even. Then you take away from the first figure the number that stands below if it is possible and, if something is left, write that in same

64 Hauksbók has “fingr” (“digit”) here.

65 “Unity is the natural starting point of all number,” from Nicomachus, *Introduction to Arithmetic*, ed. Martin Luther D’Ooge (New York: The Macmillan Company, 1926), 192.

66 Literally, “eight-cornered number” (*áttþyrnd tala*).

67 *verpill*, or, in German, *Würfel*.

place, or, else, put a cipher there. But if you cannot take away the first figure as that number which stands below is greater, then take one from the next figure and carefully note that this makes ten [added to the figure] in the first place. Then take from this the entire number as is below and [write] what remains in the same place. And if ciphers stand over above, then take one from that figure which stands next to the ciphers and write nine where the ciphers were, all the way until you come to the place where you want to take away from. And you will take from them ten as needed and write what is left in the same place.

If you want to double some number, then first write such number as you like. Next you double that figure which is farthest to the left hand and write in the next place that which remains as in addition. But if *semis*<sup>68</sup> stands over above in the outermost place, then add one since before there was an even<sup>69</sup> number which was divided in half.

But if you want to take half of a number, write such number as you want and take half of the first figure if it is even. But if it was odd, then divide in half that which remains from one less, take up the one and write over above that character which denotes half of any part,<sup>70</sup> which we call *semis* and make so ¶, and [if it is one, remove it and]<sup>71</sup> put a cipher in that place. Next take half of the second figure in the same way if it is even. But if it is odd, then take half from that which is even and just under that and make from this five, which is half of ten, in the place next to that. But if one stands in the second place, then take it up and write five in next place and put there a cipher where the one stood. A cipher does nothing unless some figure stands to the left of it. Now proceed forward in such way for as many figures as there are.

68 Latin for “a half-unit”; see Charlton Thomas Lewis and Charles Short, eds. *Latin Dictionary*. (Oxford: Clarendon Press, 1969).

69 This is clearly an error, which appears as well in Hauksbók (but not in AM 685 d 4to). The text is referring to the case in which the given number was the result of halving a previous number, resulting in a remainder of one-half. Hence the original number must have been odd (“óiofn”), not even (“iofn”).

70 “Take up the one” refers to the one removed from the initial odd number. In some texts of the Indian calculus, the remainder one is written above the final digit of the result; here, a *semis* is written instead.

71 This text is missing both in GKS 1812 and in Hauksbók but is needed to explain that this instruction applies only to the case in which the first digit is a one.

If you wish to multiply [14v] one number by another, write the two rows of characters in that way that the outermost figure of that number which you multiply stands under the first character of the upper number, and each of the others of the lower number are to the left. Next you shall consider how much the larger figure which you want to multiply lacks from ten. And as many units as are lacking from ten, that is how often you should take the smaller number, that you want to multiply, from that number of tens.<sup>72</sup> So that you understand this, multiply seven and nine. Nine is one less than ten, therefore you take one seven away from seventy, and then sixty-three remains. That is seven times nine. In the same manner you can attempt other numbers. Multiply the first figure correctly with each figure which stands under and write above each figure the multiple that it has, and to the left that which cannot be over it in the next place using correct addition. Then when this figure is multiplied, move the outermost of those that stand below under the next figure and multiply with that as with the first. And, if multiplication makes it an article, set a cipher over above and arrange the article to the left. But if both a digit and an article result from the multiplication, then write the digit over that figure which you multiplied and the article in the next place. But if only a digit results from the multiplication, then write it over above. If a cipher is in the upper number, then skip over it because nothing is a multiple of it. And also take care of this, that you remove those figures as are put above as soon as you have multiplied each, and write that digit in the place as belongs to each or a cipher if that is correct, and add that to that which stands to the left side as remains. If a cipher stands over that figure which you multiplied, then remove it if it becomes a digit after multiplication,<sup>73</sup> otherwise leave it standing in place. If you doubt whether you have multiplied correctly, then divide apart the whole multiplied number with that number which stood under. And you will get the same number as you had first.

If you want to divide apart some number, then write the two rows of the characters and write the smaller number underneath. The larger number must be two, three, or more times greater. Set the foremost figure that stands under aligned with the first above and the others to the right, continuing as long as the ones underneath last. Next think about how

72 The text does not mention for which cases this is a useful aid for multiplying digits.

73 The text has *oc margfallda*, as does Hauksbók, but *af margfalldan* would make more sense.

often the first digit also is in the upper number so that equally often those which follow [15r] it are each in that number which stands over. Set that digit aligned with the outermost figure which stands under and yet above both rows. After that take the first from the first figure and each after the other equally often from the upper number. But if one number is under, then take it from the upper number. Next move each number that stands under over by one [place] and find another quotient. Set that by the first and take the lower number as often from the upper. Do that the same way as often as needed. If you are not able to find the lower number or figure in the upper, then set the foremost digit which stands underneath next to the first and others in same way to the right and then find the quotient in such a way, and move along the figures as needed. And write each of the quotients together above so many as needed. But if a cipher stands under, then skip over it because one cannot divide by that. Then when you come under the outermost figure and have divided it, you can no longer divide and then be careful to observe that number which remains, if it is anything. If you want to prove whether you have divided correctly, then multiply that number which stood under with the quotient and you will get the same number that you had at first. And if something remained after the division, then add that afterwards to what is multiplied and you will find the same number.

When you take any number and multiply it by itself, that is called a square, or a quadrate, and the first number which you multiply is called a root. Each number is the root of some number, but not every number is a square. If you want to find the root of some number, then first write such number as you like. And in the first place, which is odd, write under it the digit which you multiplied by itself, and take from that which is above or such as comes next to it. After that you double that same digit and that is called a duple. Then take away the digit and that is called subduple. Note the subduple and write the duple in the next place if it is a digit, but if it is an article write it where the digit stood formerly and set a cipher before it, or else a digit if it is a compound number. After that find a new digit, multiply it with the duple, and take from the upper number that number which you multiplied. After that multiply the digit by itself and take that number from the upper aligned over it. Then next double the digit, note it together with the former subduple, and set the duple in the next place as

before. Then find the next new digit, multiply it with both duples at once, move the former duple [15v] towards the other duple by one place, and add it there if an article remained there from that duple. Then multiply a new digit with both the duples and take that number from the upper aligned to the duple. Do this in the same way as often as needed, multiply the new digit with all the duples, and move those always over one until you come to the outermost place. If all of that number is used up which you wrote at the beginning, then that number was a square. And the root of that number is made up of the digits all together which you doubled together with the last digit which you found. Multiply the root by itself and you will have the same number as at first if you have worked correctly. If some of the number remains after you extract the root, then that number was not a square. And you add that number to that which you multiplied the root and you will get the first number. And that number together, the root and remainder, is the root for the larger number. If the first place of that number which you wrote was even, then find a digit under the next figure and multiply in the same way.

If you multiply correctly, then multiply a square number with itself and that number which comes from this multiplication is called a *cubicus*, or cubic, number. It is the same size on all sides. Moreover, the cube root is the same as for the square number: Each number<sup>74</sup> is the root of some cubic number, or *cubicus*, but not every number is a cube.

If you want to find a cube root, note how big the number is and how many places it has. Then find the nearest digit in the farthest of the thousands place. We call places of the thousands all those which break only into thousands. That is the fourth, the seventh, the tenth, the thirteenth, and always jump over two places.

You shall start this work from the left. Multiply that digit which you found with itself cubically. That is twice multiplied, first by itself and a second time by that number which came from there. And next take from the upper number this entire number aligned with that digit itself and next triple the digit, skip over one place with that number, and set that number in the third place before it if it is a digit. But if that is an article, set there a cipher and the article in the next place. And if it is a compound number, set the digit in the same place and the article the next. Next find a new

74 Hauksbók has *tala* here, which is missing in GKS 1812.

digit in the next place, [16r] treble the number, which is called a triple, and multiply it with the figure which you found first, and which we call subtriple, and to the right of it multiply it with the triple. And then multiply it alone with that number which came from the multiplication, which we call the product. Then take this number as a whole from that upper one aligned over where the triple stood. Next multiply that same digit in itself cubically and take that number from that aligned over the digit itself. Take that digit and triple it here as the former and then find new digit. Multiply it with both the subtriple and the triple together and always move the older triple along as you do in smaller root extraction with duple, except here you shall always skip over one place and add still that same way triple to triple with correct addition. Continue in such way as long as needed and you come to the outermost place. And you shall with great care attend to, when you find the digits, that they not take so much from upper number that that number has<sup>75</sup> no place when you multiply the triple or the other numbers when you multiply the later digit. Always keep subtriple with triple. And note that if ciphers come in a subtriple, nothing is a multiple or triple of them, but they keep to their own places as long as some figure is to the right of them. And what is least difficult is the addition of a triple, so that it always goes as written before in the addition art.

All the digits together, those which were subtriples and the outermost digits too, is the root of the larger number of that which you first wrote if the subtractions used up the whole number. And you multiply the subtriples by themselves cubically and you will find the first number. But if there is some remainder to the number after subtraction, then that number is not a cube. But still the remainder, with the subtriples, forms the root of some cube. And if you multiply the root of the smaller cubically and add the remainder to that number which comes from the multiplication, you can get the first number which you wrote. And now we write at the present no more about this.

These are the digits squared: 3 squared is 9, 2 squared is 4, 4 squared is 16, 5 squared is 25, 6 squared is 36, 7 squared is 49, 8 squared is 64, 9 squared is 81. And the art to find the digits multiplied is as written before. These are the digits cubed: 3 cubed is 27, 2 cubed is 8, 4 cubed is 64, 5 cubed is 125, 6 cubed is 216, 7 cubed is 343, 8 cubed is 512, 9 cubed is 729.

75 GKS 1812 has *hafin* here, but Hauksbók has *hafí*.

Each square number has two dimensions, that is breadth and length. But a cubic number has three dimensions, that [16v] is breadth, length, and thickness or height. And therefore the sages say every visible body is put together with this number for it always<sup>76</sup> has these three dimensions.

Since eternal wisdom, the one God wanted to create a visible and bodily world, then he first made two of the outermost elements, fire and earth, because nothing can be naturally visible without them, as fire makes light and movement, but the earth is steadfast and unmoving. And since they have three unequal and opposite qualities, it was then a natural necessity to set something between them which reconciled their differences. And since, as said before, fire and earth, and every such thing that is of a bodily nature is composed out of a three-fold number, which we call cubic, then we write for this two cubes. We write the earth this way: twice times two twice, 2, 4, 8. But fire so: thrice three thrice, 3, 9, 27.

And since in this case no single mean<sup>77</sup> can be between these numbers, with only it being in equal proportion to both and of no other two cubes, then we find two proportional numbers in this manner: We multiply the root of the greater cube with the square of the smaller cube, that is twice two thrice: 2, 4, 12. And the root of the smaller cube with the square of the greater cube, that is thrice three twice: 3, 9, 18. These two numbers belong equally to the outer two cubes since seven and 20 has in itself 18 and half of 18, and 18 has in itself 12 and half of 12. And so has 12 in itself 8 and half of 8. You shall in the same way always find proportions between two cubes.

Thus God arranged two elements between fire and earth: air and water. And water has two qualities from earth and two numbers, but from fire one quality and one number. And air has two qualities from fire and two numbers, but one from earth and one number. And fire is as much lighter than air as 27 is greater than 18. And air is as much lighter than water as 18 is greater than 12. Water is as much lighter than earth as 12 is greater than 8.

This is to be more fully understood in that figure which is made here later and is called the *cubus perfectus*.<sup>78</sup>

<sup>76</sup> GKS 1812 has *jamnan* here, but Hauksbók has *avallt*.

<sup>77</sup> *miðskeið*

<sup>78</sup> This work is derived from my MA thesis in Medieval Icelandic Studies at the University of Iceland. I would like to thank Gunnar Harðarson for his encouragement, suggestions, and careful reading of the transcript and translation. His corrections have greatly improved

## BIBLIOGRAPHY

## MANUSCRIPTS

*Stofnun Árna Magnússonar í íslenskum fræðum, Reykjavík*

GKS 1812 4to, 13v-16v. [*handrit.is*: <https://handrit.is/en/manuscript/view/is/GKS04-1812>; *digitalesamlinger.hum.ku.dk*: <http://digitalsamlinger.hum.ku.dk/Home/Samlingerne/33608>]

*Den Arnamagnæanske Samling, Institut for Nordiske Studier og Sprogvidenskab, Københavns Universitet, Copenhagen*

AM 544 4to, 90r-93r. [*handrit.is*: <https://handrit.is/en/manuscript/view/en/AM04-0544>]

AM 685 d 4to, 24v-29r. [*digitalesamlinger.hum.ku.dk*: <http://digitalsamlinger.hum.ku.dk/Home/Samlingerne/8043>]

AM 736 III 4to, 4r-4v. [*digitalesamlinger.hum.ku.dk*: <http://digitalsamlinger.hum.ku.dk/Home/Details/148601>]

## PRIMARY SOURCES

*Hauksbók*. Ed. by Finnur Jónsson and E[iríkur]. Jónsson. Copenhagen: Thieles Bogtrykkeri, 1892–96.

*Muḥammed ibn Mūsā al-Khwārizmī: Le calcul indien (Algorismus)*. Ed. by André Allard. Namur, Belgium: Société des Études Classiques, 1992.

Nicomachus. *Introduction to Arithmetic*. Ed. by Martin Luther D'Ooge. New York: The Macmillan Company, 1926.

Petrus de Dacia. *Petri Philomeni de Dacia In Algorismum Vulgarem Johannis de Sacrobosco Commentarius*. Ed. by Maximilianus Curtze. Copenhagen: A. F. Host & Fil. Bibliop. Reg., 1897.

— — —. *Petri Philomenae de Dacia et Petri de S. Audomaro Opera quadrivialia*. Ed. by Fridericus Saaby Pedersen. Copenhagen: Societas Linguae & Litterarum Danicarum, 1983.

*Rara Mathematica*. Ed. by James Orchard Halliwell. London: John William Parker, 1839.

William of Conches. *Guillelmi de Conchis: Glosae Super Platonem*. Ed. by Édouard A. Jeauneau. Turnhout: Brepols, 2006.

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## SECONDARY SOURCES

- Allard, André. "The Influence of Arabic Mathematics in the Medieval West." *Encyclopedia of the History of Arabic Science*, ed. by Roshdi Rashed. London: Taylor & Francis, 1996, 2:514–55.
- "AM 544 4to: Hauksbók; Iceland and Norway, 1290–1360." <https://handrit.is/en/manuscript/view/en/AM04-0544>.
- "AM 685 d 4to." *ONP: Ordbog over det norrøne prosasprog*. Copenhagen: Den Arnamagnæanske Kommission. <https://onp.ku.dk/onp/onp.php?m135>.
- "AM 736 III 4to." *ONP: Ordbog over det norrøne prosasprog*. Copenhagen: Den Arnamagnæanske Kommission. <https://onp.ku.dk/onp/onp.php?m136>.
- Bekken, Otto B. "Algorismus of 'Hauksbók': An Old Norse Text of 1310 on Hindu-Arabic Numeration and Calculation." *Agder: Agder distrikthøgskole*, 1986. <https://web.archive.org/web/20070928032455/http://home.hia.no/~ottob/ALGORISMUS.pdf>
- Bekken, Otto B., and Marit Christoffersen. "Algorismus." *Medieval Scandinavia: An Encyclopedia*, ed. by Phillip Pulsiano and Kirsten Wolf. New York: Routledge, 2016, 8–9.
- Bekken, Otto B., Marit A. Nielsen, and Steinar Thorvaldsen. *Algorismus i Hauksbók: En norrøn regnetekst fra 1300-tallet*. Tromsø: Eureka Digital, 2010.
- Benedict, Suzan Rose. *A Comparative Study of the Early Treatises Introducing into Europe the Hindu Art of Reckoning*. Concord: The Rumford Press, 1914.
- Bouzari, Abdelmalek. "The Calculus of Al-Khwārizmī." *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, ed. by Gunnar Harðarson with Christian Etheridge, Guðrún Nordal, and Svanhildur Óskarsdóttir. Forthcoming, 155–69.
- Folkerts, Menso. "Early Texts on Hindu-Arabic Calculation." *Science in Context* 14, nos. 1-2 (2002): 13–38.
- Gunnar Harðarson. "Medieval Encyclopedic Literature and Icelandic Manuscripts." *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, ed. by Gunnar Harðarson with Christian Etheridge, Guðrún Nordal, and Svanhildur Óskarsdóttir. Forthcoming, 13–38.
- Gunnar Harðarson and Stefán Karlsson. "Hauksbók." *Medieval Scandinavia: An Encyclopedia*, ed. by Phillip Pulsiano and Kirsten Wolf. New York: Routledge, 2016, 271–72.
- Hagland, Jan Ragnar. *Riksstyring og språknorm: Spørsmålet om kongens kanselli i norsk språkhistorie på 1200- og første halvdel av 1300-talet*. Dragvoll, 1984.
- Haraldur Bernharðsson. "Scribes and Scribal Practice in GKS 1812 4to." *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, ed. by Gunnar Harðarson with Christian Etheridge, Guðrún Nordal, and Svanhildur Óskarsdóttir. Forthcoming, 63–113.
- Kristín Bjarnadóttir. "Algorismus: Fornt stærðfræðirit í íslenskum handritum."

- NETLA (2004). <http://wayback.vefsafn.is/wayback/20201017180549/>  
<https://netla.hi.is/greinar/2004/001/index.htm>.
- Kristín Bjarnadóttir and Bjarni V. Halldórsson. “Algorismus: Hindu-Arabic Arithmetic in GKS 1812 4to.” *A World in Fragments: Studies on the Encyclopedic Manuscript GKS 1812 4to*, ed. by Gunnar Harðarson with Christian Etheridge, Guðrún Nordal, and Svanhildur Óskarsdóttir. Forthcoming, 171–91.
- – –. “The Norse Treatise Algorismus.” *Actes du 10ème Colloque maghrébin sur l’histoire des mathématiques arabes*. Tunis: Association Tunisienne des Sciences Mathématiques, 2011.
- – –. “Ritgerðin Algorismus – samanburður handrita.” *Vísindavefur: Ritgerðasafn til heiðurs Þorsteini Vilhjálmsyni sjötugum 27. september 2010*. Reykjavík: Hið íslenska bókmenntafélag, 2010.
- Lewis, Charlton Thomas, and Charles Short, eds. *Latin Dictionary*. Oxford: Clarendon Press, 1969.
- “Menota Handbook 3.o.” *Menota Nordic Text Archive*. Menota, 2019. [https://www.menota.org/HB3\\_ch4.xml](https://www.menota.org/HB3_ch4.xml).
- Munch, P.A. “Algorismus, eller Anviisning til at kjende og anvende de saakaldte arabiske Tal, efter Hr. Hauk Erlendssons Codex.” *Annaler for nordisk oldkyndighed og historie*, (1848): 353–75.
- Seip, Didrik Arup. *Norsk språkhistorie til omkring 1370*. 2. utgave. Oslo: Aschehoug, 1955.
- – –. *Palæografi: B Norge og Island*. Nordisk kultur 28B. Stockholm: Albert Bonniers forlag, 1954.
- Smith, David Eugene, and Louis Charles Karpinski. *The Hindu-Arabic Numerals*. Boston: Ginn & Company, 1911.
- Vågslid [=Vågslid], Eivind. *Norske logmannsbrev frå millomalderen: Ei skrifthistorisk etterrøking av logmannsbrev frå Oslo, Uppland, Skien, Tunsberg, Borgarting og Bohuslän*. Oslo: Det Norske Videnskaps-Akademi i Oslo, 1930.

## ÁGRIP

*Algorismus* í GKS 1812 4to: Uppskrift og þýðing

**Efnisorð:** *Algorismus*, *Carmen de Algorismo*, GKS 1812 4to, Hauksbók, stafrétt útgáfa

Í þessari grein er birt stafrétt útgáfa þeirrar gerðar *Algorismus*, norrænnar þýðingar á *Carmen de Algorismo* eftir Alexander de Villadei (um 1170–um 1240), sem er að finna í alfræðihandritinu GKS 1812 4to 13v1–16v31. Sú gerð hefur ekki áður verið prentuð en hún hefur ýmis sérkenni, einkum skriftarfræðileg, svo og nokkur lesbrigði, sem greina hana frá þeirri gerð sem áður hefur verið gefin út og byggist á AM 544 4to sem er hluti Hauksbókar. Einnig er hér birt fyrsta enska þýðingin á *Algorismus* svo vitað sé.

## SUMMARY

*Algorismus* in GKS 1812 4to: Transcription and Translation

**Keywords:** *Algorismus*, *Carmen de Algorismo*, GKS 1812, Hauksbók, diplomatic edition

This article provides a diplomatic edition of the *Algorismus*, an Old Norse translation of the *Carmen de Algorismo* of Alexander of Villedieu (c. 1170–c. 1240), found in the encyclopedic manuscript GKS 1812 4to 13v1–16v31. This version has not been published previously. It has various characteristics, most notably in the script as well as in some readings, which distinguish it from previous editions that were based on AM 544 4to, a part of Hauksbók. Also, included here is, as far as is known, the first English translation of the *Algorismus*.

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